BOOK REVIEW: B. ØKSENDAL, A. SULEM, *APPLIED STOCHASTIC CONTROL OF JUMP DIFFUSIONS*

SUBRAMANIAN RAMAMOORTHY

The notion of optimality plays a central role in a variety of application domains, making optimal control theory an essential part of the toolbox of engineers and applied scientists. This book by Øksendal and Sulem is aimed at introducing these applied scientists to techniques for the optimal control of a specific type of process - jump diffusions, solutions of stochastic differential equations driven by Levy processes (which, in turn, may be understood as a linear combination of drift, diffusion and pure jump terms).

This is an applied book in the sense that, although all of the arguments are stated in formal probabilistic language, the focus is not on the intricacies of proofs (which are either only sketched or omitted altogether by citing specialist literature). This allows the book to be short and better focussed on how one might use the theory and what one gains from this. The ideas are developed by showing how one should adapt well known machinery, such as the Hamilton-Jacobi-Bellman sufficient condition, Pontryagin Maximum Principle or variational inequalities, to the case of jump diffusions. So, many of the earlier chapters work through the mechanics of re-deriving approximation or verification theorems for the case where the stochastic process includes jumps. The more important results in this book involve the definition of Hamilton-Jacobi-Bellman Variational Inequalities (HJBVI) that generalize both the stochastic control problem with fixed stopping time and the variational inequalities corresponding to optimal stopping problems. This can be taken even further by defining singular and impulse control schemes, which in the most general cases may result in HJBQVIs admitting only weak viscosity solutions.

The book is a research monograph that requires a firm background in measure-theoretic probability and the mathematical theory of stochastic optimal control (or, at least, the rigorous deterministic version). Readers with this background will find the discussion engagingly brisk. The authors do a very nice job of showing why these ideas matter in financial problems, e.g., their examples on the jump-diffusion version of the Black-Scholes model, mean-variance portfolio selection, dividend policy, etc. are all very nicely done and clearly illustrate, with a few figures serving as helpful aids, the underlying conceptual ideas. The book includes many worked examples (and several more unsolved exercises) that will serve the dedicated student in good stead. However, with a few scattered exceptions, the applied focus of this book is entirely on computational finance. Readers who come to this book with no prior exposure to financial or economic problems and with different domain interests may well find this distracting.

Although this is an applied book, there is only limited discussion of practical matters such as numerical solutions of the stochastic differential equations. In the final few pages of the book, the authors sketch a finite difference scheme for these processes and a policy iteration algorithm. Beyond that, it is expected that the reader will be able to bootstrap from their prior knowledge of numerical and computational methods to this extended setting. A more fundamental and conceptual issue that isn't really discussed is the robustness of the solutions in the face of model mis-specification. In the current climate faced by practitioners of computational finance, one might wonder about the sensitivity of the singular and impulse control schemes to errors in modeling and estimation. However, this may well be more of a research question beyond the scope of the current monograph.

In summary, this is a good and relatively inexpensive book that should appeal to graduate students and researchers with some prior knowledge of stochastic control who now wish to learn about the jump diffusion case - especially, as applied in the areas of computational finance and economics.

School of Informatics, The University of Edinburgh, Informatics Forum, 10 Crichton Street, Edinburgh EH8 9AB, UK

E-mail address: s.ramamoorthy@ed.ac.uk